Maximov V., Kornoushenko E. (2001). Mathematical Basics of Construction the Graph and Computer Models for Complicated Situations. Proceedings of the IFAC Symposium on Modelling and Control of Economic Systems (SME 2001).

MATHEMATICAL BASICS OF CONSTRUCTION THE GRAPH AND COMPUTER MODELS FOR COMPLICATED SITUATIONS

Maximov V., Kornoushenko E.

Maximov V. - Institute of Control Sciences of Russian Academy of Sciences,
Head of Sector of Cognitive Analysis and Situation Modelling, Ph.D.;
Kornoushenko E. - Institute of Control Sciences of Russian Academy of Sciences,
Head of Sector of Dynamic Systems Diagnostics, Dr.Sci.

Abstract: The analytical approach to the analysis and control of processes in situations with lack of the sufficient numerical information about their internal processes is offered. This approach may be applied for solution of economic, social, political, ecological, and other problems, taking into account that the construction of exact numerical models of complex social situations is hampered or even impossible by virtue of lack of required numerical information.

Keywords: Cognitive science, cognitive systems, economic systems, forecasts, modelling

1. INTRODUCTION

Economic, political, ecological, and other social systems are characterized with lack of detailed quantitative information about their internal processes, as opposed to engineering systems. The information in social systems has, as a rule, qualitative character. When solving a control problem in these systems, at first the researcher has to structure available information and to extract leading parameters driving the system activity.

Cognitive structuring, which promotes the best understanding of originated problems, detection of inconsistencies and qualitative analysis of the economic, social or political system, seems to be a convenient tool for research of these systems, as this was shown yet in F. Roberts' (1976) book. Practical achievements of the last years in the field of intelligent technologies have created a favourable basis, so that the cognitive paradigm became attractive and popular. Now it promptly wins wide gratitude among the experts in the field of business and economic management. In the other article of authors (Kornoushenko and Maximov, in this issue) to be included in these Proceedings, it was shown that cognitive approach should be used when solving a control problem for a system to provide its goal-seeking behavior depending essentially on system parameters interference.

The knowledge acquisition process may be divided into more "thin" processes (extraction, acquisition, shaping) having its own specificity. During extraction of knowledge there is an interaction of the expert ("source" of knowledge) and the cognitologist ("modelist"). Their interaction allows to trace a course of reasoning of the experts at decision-making and to reveal structure of their representations about a problem domain. As a result of their co-operation, the cognitologist creates a "frame" model of a problem domain using his experience in cognitive psychology, system analysis, mathematical logic, etc. Structuring of knowledge about system under consideration includes creation of list of basic concepts, determination of the relations between them, setting of goals and determination of initiatives to be realised to achieve the goals. In other words, knowledge about a system may be represented as a weighted oriented graph, table, text etc.

Selection of the basic concepts (factors) is carried out on the basis of PEST-analysis (Politics, Economy, Society, Technology), by means of which the political, economic, social, cultural, and technological aspects of system under study and of its external environment are analysed. Therefore, it may be convenient to consider a pair “system + interface of its external environment” as a complicated “situation”. Thus, for the situation under study a special set of the key factors are extracted, which directly and most significantly influences it. The analysis of each of the allocated aspects is carried out systematically, as far as all these aspects are closely and intricately interconnected. The significant change of any of aspects, as a rule, influences all chain. Such changes in each concrete case can create a threat to the system development, or, on the contrary, a new strategic opportunity of its future successful development.

2. CONSTRUCTION OF COGNITIVE MODEL OF COMPLEX SITUATION

The cognitive map of a situation is represented as an oriented weighed graph, in which

- Nodes correspond one to one to the basic factors of a situation in terms of which the processes in a situation are described. The set of originally selected basic factors may be verified with the help of a data mining process, permitting to reject the “surplus” factors “poorly connected” to “kernel” of the basic factors;

- The direct correlation between the factors is determined by reviewing the influences from each factor to the others. It is considered that the factors included in premise “if...” of the statement “if..., then...” influence the factors of a corollary “that...” of this statement and this influence may be either strengthening (positive), or breaking (negative), or variable one depending on possible side conditions.

- The arc *(i, j),* drawn from node *i* to node *j*, depicts the fact that the change of values of the factor V_i is followed by change of values of the factor V_j . Arc *(i, j)* has a sign (+), while this influence "positively" and sign (-) in an opposite case. A degree of such influence is represented using linguistic variables with values from an interval [0,1]: 0.1 — "very weak"; 0.3 — "moderate"; 0.5 — "essential"; 0.7 — "strong"; 1.0 — "very strong".

Thus, weight $a_{ij} \in [-1,1]$) is added to each arc *(i, j).* As a result, a situation model is obtained as the weighted digraph $G = (V, A)$, where V is a node set (basic factors set), and A is a set of the weighted arcs.

Despite of numerical values of arc weights, graph model is also qualitative by virtue of the fact that linguistic variables are used in its construction. To simulate the dynamics of its internal processes, the concept of time is necessary to add in graph model. The computer modeling of processes are carried out in discrete time with sampling time interval of the chosen dimension (second, hour, day, week, month, quarter, etc).

Let $V_i(t)$ be a trend of factor with number *i* and time *t* and let this trend be measured in some linguistic scale. The qualitatively measured trend V_i may be interpreted as qualitative “derivative” of factor *i*. The basic assumption to graph model dynamics is that an increment $V_i(t+1) - V_i(t)$ of the trend V_i at the time $t+1$ linearly depends on increments of trends of “adjacent" factors acting on factor *i* at the previous time *t*. This dependence is represented as

$$V_i(t+1) = V_i(t) + \sum_{j \in I_i} a_{ij}\left(V_j(t) - V_j(t-1)\right) + g_i(t),\ i = 1,\dots,n \tag{1}$$

where I_i is a node set in the graph G such that each node out of this set is connected by arc to factor *i* and $g_i(t)$ is a possible external control to be supplied to node *i* at time *t*. If $g_i(t) \neq 0$ for some *t*, factor *i* refers to as controlling one**.**

Despite of simplicity of model (1), it is widely applied to the analysis of complex systems in framework of signed and weighed directed graphs (see, for example, Kornoushenko E. and Maximov V. (In: this issue.)).

The set of relations (1) for all $i=1, \dots, n$ may be represented in a matrix form:

$$V(t+1) = (E_n + A)V(t) - AV(t-1) + Bg(t), \tag{2}$$

where $V(t) = (V_1(t), \dots, V_n(t))^T$, T — transposition, A be a transposed and “weighted” adjacent matrix of the graph G, E_n an identity matrix of order n, $g(t) = (g_1(t), \dots, g_p(t))$ a control vector, and matrix B specifies controlling nodes in the graph G. In according to (2), graph nodes set and controlling nodes are defined a priori and do not vary in the course of modeling (though, by the way, for some instants some coordinates of a vector $g(t)$ can be equal to zero).

The expression (2) defines, as a matter of fact, a system of the linear finite-difference equations. Let $V(0)$ represents an initial state of a situation (in terms of its factors trends). Using model (2), it is possible to find a state $V(t)$ of a situation at any time *t* for any control sequence $g(0), g(1), \dots, g(t-1)$:

$$V(t) = (\sum_{i=0}^{t} A^{t}) V(0) + \sum_{k=0}^{t-1} \sum_{j=0}^{t-k-1} (A^{T}) Bg(k). \quad (3)$$

Note 1. It is possible that some coordinates of vectors $V(t_1)$, $V(t_2)$...will increase unboundedly due to instability of matrix A. However, for plausible interpretation of modeling results it is desirable that matrix A be stable. This may be done with division of each *i-th* row (or *i-th* column) of a matrix A by value N_i, where N_i be the number of nonzero elements of *i-th* row (or *i-th* column), $i = 1,..., N$ and N an order of matrix A. Modified matrix A is denoted as A^*.

In this case equation (2) is substituted by the following one:

$$V(t+1) = (E_n + A^{*}) V(t) - A^{*} V(t-1) + Bg(t) \quad (4)$$

The factors to be interested for and observed by researcher are called as output ones. A vector $y(t)$ of observable factors is defined as:

$$y(t) = CV(t) \quad (5)$$

where $y(t) = (y_{J_1}(t),..., y_{Jm}(t))^T$ be an output vector and matrix C specifies the output factors set. The relations (4) - (5) define computer model of a situation.

Note 2. By virtue of the fact that each factor and each control have corresponding subject interpretation in realistic situation, therefore:

- The supply of several controls of different modalities at the same controlling node of the graph G is inadmissible;
- Any control should not be supplied to several nodes;
- The addition of factors values of different modalities is inadmissible.

Therefore each row and each column of matrixes B and C have to contain no more than one unit.

At the analysis of realistic situation the user usually knows or assumes, what trends of some basic factors are desirable for him. These factors are user's goal factors. The solution of control problem in a situation under study is to provide desirable trends of the user's goal factors, it is the core of control problem. The goal is considered to be correctly preassigned, if the desirable trend of some goal factor does not cause undesirable trends of other goal factors.

In the set of basic factors the subset of the so-called controlling factors ("input" factors of the cognitive model) is selected. Control actions in the model are realized via these factors. The control action is considered to be co-ordinated with the goal, if it does not cause undesirable trend in any of the goal factors.

If the goal of control is preassigned correctly and the control actions are co-ordinated with this goal, the solution of a control problem does not cause any specific difficulties (even in non-linear cognitive model of a situation with constant signs of factor influences). In a common case the determination of conditions of goal-seeking behaviour in a situation is far not a simple problem. In view of note 2, any changes of the basis in the space of basic factors are inadmissible. This fact makes impossible using the results of modern control theory to construct a regulator with prescribed properties for situation under study. Alternative approach to solution of control problem is proposed below.

3. DETERMINATION OF CONTROLS TO ACHIEVE PREASSIGNED GOALS

In applications of proposed model to realistic situations the following problem is of large interest: to find the controls transferring a situation initial state $V(0)$ to some state, such that trends of goal factors in this state become close (or equal) to the preassigned chosen "goal" trends. This problem is known as an inverse control problem. A solution of this problem is described under following reductive assumptions:

a) The controls have to be of impulse character and are supplied to controlling nodes in an initial time $t = 0$ only;

b) The goal factors steady trends should be close (or equal) to chosen "goal" trends of these factors.

Let $y^*(t) = (y^*_{J_1}(t),..., y^*_{Jm}(t))^T$ be a chosen "goal" trends vector. Taking into consideration assumptions a) and b), we obtained from (3) and (5):

$$y(\infty) = C\left[\sum_{i=0}^{\infty} (A^{*})^{i}\right](V(0) + Bg(0)) = y^* \quad (6)$$

The expression (6) represents the system of m linear algebraic equations for searching coordinates $(g_{J_1}(t),..., g_{Jp}(t))^T$ of vectors $g(0)$.

Since the matrix A^* is stable it is true:

$$\sum_{i=0}^{\infty} (A^*)^{i} = (E_n - A^*)^{-1}$$

Then the equation with respect to unknown vector $g(0)$ is as follows

$$C(E_n - A^*)^{-1} B g(0) = y^* - C(E_n - A^*)^{-1} V(0) \quad (7)$$

Transfer function $W(z)$ from controlling nodes to goal nodes is:

$$W(z) = C(zE_n - A^*)^{-1} B$$

Then $C(E_n - A^*)^{-1} B = W(1)$ and equation (6) may be note more compactly:

$$W(1) g(0) = \tilde{y}, \quad (8)$$

where $\tilde{y} = y^* - C(E_n - A^*)^{-1} V(0)$.

If $p > m$, the set of equations (7) is undetermined and has a uncountable set of exact solutions, defined according to the least square method by formula:

$$g(0) = W^+(1)\tilde{y} + (E_p - W^+(1)W(1))h \quad (9)$$

Here $W^+(1)$ is a pseudoinverse matrix for a ($m \times p$)-matrix $W(1)$and h is an arbitrary vector with p coordinates.

Note 3. In principle, any control vector to be chosen according to (9) will ensure fitness of goal factors steady trends to goal factors preassigned trends, so there is a rather large set of suitable controls for a choice. We will be interested with normal solution of a system (8)

$$g(0) = W^+(1)\tilde{y}.$$

As it was shown in Kornoushenko E. and Maximov V. (In: this issue), the cognitive models may be used to solve control problems in various complex situations, in particular for correction and tactical management of enterprise activity.

4. CONCLUSION

The analytical approach to the analysis and control of processes in situations with lack of the sufficient numerical information about their internal processes is offered. This approach may be applied for solution of economic, social, political, ecological, and other problems, taking into account that the construction of exact numerical models of complex social situations is hampered or even impossible by virtue of lack of required numerical information.

The approach represented herein was applied successfully to solution of problems of regional development, manufacturing development, etc. The results of these works may be seen on http://www.ipu.ru/labs/lab51/51_home.htm.

REFERENCES

Kornoushenko E., Maximov V. Cognitive model for correction and tactical management of enterprise activity. *In: this issue.*

Roberts F.S. (1976). *Discrete mathematical models with applications to social, biological and environmental problems.* Prentice-Hall, Inc., Englewood Cliffs, N.J.

http://www.ipu.ru/labs/lab51/51_home.htm