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ANALYTICAL BASICS OF CONSTRUCTION THE GRAPH AND COMPUTER MODELS FOR COMPLICATED SITUATIONS

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Abstract: The analytical approach to the analysis and control of processes in complex situations with poor numerical information about their states is offered. This approach may be applied for solution of many problems (economic, social, political, ecological, etc), taking into account that the construction of exact numerical models of such situations is hampered or even impossible by virtue of lack of required numerical information.

Keywords: socio-economic systems, cognitive model, inverse control problem, modeling.

1. INTRODUCTION

Cognitive structuring, which promotes the best understanding of originated problems, detection of inconsistencies and qualitative analysis of the economic, social or political systems, seems to be a convenient tool for research of these systems, as this was shown yet in F. Roberts’ (1976) book. Practical achievements of the last years in the field of intelligent technologies have created a favourable basis, so that the cognitive paradigm became attractive and popular. Now it promptly wins wide gratitude among the experts in the field of business and economic management.

Structuring of knowledge about any socio-economic system under consideration includes creation of list of basic concepts, determination of the relations between them, setting of goals and determination of initiatives to be realised to achieve the goals. In other words, knowledge about a system may be represented as a weighted oriented graph, table, text etc. Selection of the basic concepts (factors) is carried out on the basis of PEST-analysis (Politics, Economy, Society, Technology), by means of which the political, economic, social, cultural, and technological aspects of system under study and of its external environment are analysed. Therefore, it may be convenient to consider a pair “system + its external environment” as a complicated “situation”. Thus, for the situation under study a special set of the key factors are extracted, which directly and most significantly influences it. The analysis of each of the allocated aspects is carried out systematically, as far as all these aspects are closely and intricately interconnected. The significant change of any of aspects, as a rule, influences all chain. Such changes in each concrete case can create a threat to the system development, or, on the contrary, a new strategic opportunity of its future successful development.

2. TWO CLASSES OF COGNITIVE MODELS OF COMPLEX SITUATION

2.1. Situation cognitive map

Situation cognitive map is represented as an oriented weighted graph, in which

• Nodes correspond one to one to the basic factors of a situation in terms of which the processes in a situation are described. The set of originally selected basic factors may be verified with the help of a data mining process, permitting to reject the "surplus" factors "poorly connected" to "kernel" of the basic factors;

• The direct correlation between the factors is determined by reviewing the influences from each factor to the others. It is considered that the factors included in premise "if..." of the statement "if..., then..." influence the factors of a corollary "that..." of this statement and this influences may be either strengthening (positive), or breaking (negative), or variable one depending on possible side conditions. Each direct influence from some factor to another one is stated on the basis of:

- corresponding economic or social well-known law,
- empirical dependencies if these dependencies for given pair of factors were stated and verified,
- expert conclusions.

• The arc (i, j), drawn from node i to node j, depicts the fact that the change of values of the factor Vi is followed by change of values of the factor Vj. Arc (i, j) has a sign (+), while this influence "positively" and sign (-) in an opposite case. A degree of such influence is represented using linguistic variables with values from an interval [0,1]: 0.1 — "very weak"; 0.3 — "moderate"; 0.5 — "essential"; 0,7 — "strong"; 1.0 — "very strong".

Thus, weight $a_{ij} \in [-1,1]$ is added to each arc (i, j). As a result, we obtain situation cognitive map as weighted digraph G = (V, A), where V is a node set (basic factors set), and A is a set of the weighted arcs. Despite of numerical values of arc weights, graph model is also qualitative by virtue of the fact that linguistic variables are used in its construction.

2.2. Situation cognitive model based on factor trends (model of class A)

To simulate the factor dynamics, the concept of time is necessary to add in situation graph model. The computer modeling of processes are carried out in discrete time with sampling time interval of the chosen dimension (second, hour, day, week, month, quarter, etc).

Let Vi (t) be a trend of factor with number i and time t and let this trend be measured in some linguistic scale. The qualitatively measured trend Vi may be interpreted as qualitative "derivative" of factor i. The basic assumption to graph model dynamics is that an increment Vi(t+1) -Vi (t) of the trend Vi at the time t+1 linearly depends on increments of trends of "adjacent" factors acting on factor *i* at the previous time t. This dependence is represented as

$$V_i(t+1) = V_i(t) + \sum_{j \in I_i} a_{ij}\big(V_j(t) - V_j(t-1)\big) + g_i(t),\ i = 1,...,n \qquad (1)$$

where Ii is a node set in the graph G such that each node out of this set is connected by arc to factor i and gi (t) is a possible external control to be supplied to node i at time t.

Despite of simplicity of model (1), it is widely applied to the analysis of complex systems in framework of signed and weighed directed graphs (see, for example, [1]).

The set of relations (1) for all i=1,…,n may be represented in a matrix form:

$$V(t+1) = (E_n + A)V(t) - AV(t-1) + Bg(t), \qquad (2)$$

where A be a transposed and "weighted" adjacent matrix of the graph G, En an identity matrix of order n, g(t) a control vector, and matrix B specifies controlling nodes in the graph G. In according to (2), graph nodes set and controlling nodes are defined a priori and do not vary in the course of modeling (though, by the way, for some instants some coordinates of a vector g (t) can be equal to zero).

The expression (2) defines, as a matter of fact, a system of the linear finite-difference equations. Let V(0) represents an initial state of a situation (in terms of its factors trends). Using model (2), it is possible to find a state V (t) of a situation at any time t for any control sequence g (0), g (1), ... , g (t-1):

$$V(t) = (\sum_{i=0}^{t} A^t)V(0) + \sum_{k=0}^{t-1} \sum_{j=0}^{t-k-1} (A^T)Bg(k). \qquad (3)$$

Note 1. It is possible that some coordinates of vectors V (t1), V (t2)...will increase unboundedly due to instability of matrix A. However, for plausible interpretation of modeling results it is desirable that matrix A be stable. This may be done with division of each i-th row (or i-th column) of a matrix A by value N i , where N i be the number of nonzero elements of i-th row (or i-th column), i = 1,…, N and N an order of matrix A. Modified matrix A is denoted as A*.

In this case equation (2) is substituted by the following one:

$$V(t+1) = (E_n + A^*)V(t) - A^* V(t-1) + Bg(t) \quad (4)$$

The factors to be interested for and observed by researcher are called as output ones. A vector *y (t)* of observable factors is defined as:

$$y(t) = CV(t) \quad (5)$$

where *y(t)* be an output vector and matrix *C* specifies the output factors set. The relations (4) - (5) define situation cognitive model based on factor trends (model of class A).

2.3. Situation cognitive model based on factor states and trends (model of class B)

Under modeling of processes in realistic situation it is often necessary take into account both the current trends and states of factors. To construct situation cognitive model of class B, first of all it is necessary to define the notion "a state" of model. We accept the following assumptions.

Assumption 1. a) Let Δt be the length of some time intervals such that current information about the situation is disposable to the analyst at boundary points of these intervals only.

b) At any time some "reference" value may be correlated to any basic factor and current value of this factor is compared with this "reference" value. Current qualitative state of this factor is defined as some function of current value of this factor and corresponding "reference" (for example, planned) value. Such function may be relative deviation from planned value (from moving average, etc). Some scale is chosen for representation the factor qualitative values.

c) Trend of factor *V* at time *t* is defined in a natural way as $\Delta V(t) = V(t) - V(t-1)$, where *V(t)* is the qualitative value of factor *V* at time *t*.

A cognitive model of class B has the form

$$V(t+1) = V(t) + A^* \Delta V(t) + Bg(t), \quad (6)$$

$$y(t) = C(t)\, V(t), \quad (7)$$

where matrix *A** is the same as in (4). It is necessary to note that some constraints may be imposed on qualitative factor admissible values. Particularly, the most frequent constraint is non-negativity of qualitative values, i.e.

$$V(t) \geq 0. \quad (8)$$

Subject to (8), models of class B are, as a rule, nonlinear.

Note 2. Cognitive model (6)-(8) is a structured and simplified dynamic description of situation under consideration. In addition, it is unlikely that its equations represent directly the time dependencies between various factors even in qualitative form. However, the signs of influences from any factors to another ones may hold true on chosen horizon. It is the assumption that enables one to use this model for situation analysis and control. Keeping these direction signs in the mind, the analyst may choose admissible initiative (scenario) and calculate the consequences of his actions. It is such possibility that enables one to say about practical availability of proposed models.

3. DETERMINATION OF CONTROLS TO ACHIEVE PREASSIGNED GOALS

3.1. Definition of goal and controlling factors

At the analysis of realistic situation a user usually knows or assumes what trends (or states, or both ones) of some basic factors are desirable for him. These factors are user's goal factors. The solution of control problem in a situation under study is to provide desirable trends (or states, or both ones) of the user's goal factors, it is the core of control problem.

In the set of basic factors the subset of the so-called controlling factors (user's "leverage" factors) is selected. User's control actions in the model are realized via these factors. . If current values of goal coordinates are closed to corresponding preassigned values in goal vector, the user supposes that a situation evolves in "right" direction and he hasn't to intervene in its evolution. But in opposite case he has to change the direction of evolution with use of appropriate control vector.

3.2. Inverse control problem solution using model of class A

Control problem using model of class A is the following: to find the controls transferring a situation initial state *V (0)* to some state, such that trends of goal factors in this state become close (or equal) to the preassigned chosen "goal" trends. This problem is known as an inverse control problem. A solution of this problem is described under following reductive assumptions:

a) The controls have to be of impulse character and are supplied to controlling nodes in an initial time $t = 0$ only;

b) The goal factors steady trends should be close (or equal) to chosen "goal" trends of these factors.

Let y^* be a chosen "goal" trends vector. Taking into consideration assumptions a) and b), we obtained from (3) and (5):

$$y(\infty) = C\left[\sum_{i=0}^{\infty}(A^*)^i\right](V(0) + Bg(0)) = y^*. \quad (9)$$

The expression (9) represents the system of m linear algebraic equations for searching

coordinates of vector $g(0)$.

Since the matrix A^* is stable it is true:

$$\sum_{i=0}^{\infty}(A^*)^i = (E_n - A^*)^{-1}$$

Then the equation with respect to unknown vector $g(0)$ is as follows

$$C(E_n - A^*)^{-1}Bg(0) = y^* - C(E_n - A^*)^{-1}V(0) \quad (10)$$

If dim(g)>dim(y), the solution of underdetermined system (10) obtaining with use the least square method is

$$g^*(0) = W^+(y^* - C(E_n - A^*)^{-1}V(0)) \quad (11)$$

where $W = C(E_n - A^*)^{-1}B$ and W^+ be pseudoinverse for matrix W. Vector g*(0) is an inverse control problem solution.

3.3. Inverse control problem solution using model of class B.

Let $V(0)$ and $\Delta V(0)$ be an initial qualitative state and initial trend vector correspondingly and let absolute values of current trends considerable less than corresponding coordinate values. Keeping note 2 in mind and without using any control factors, in "linear" considering we obtain subject to (6):

$$V(t) = V(0) + (\textstyle\sum_{k=1}^{t}(A^*)^k)\Delta V(0).$$

So, the initial state V(0) is transferred to steady state V_{st}

$$V_{st} = V(0) + (\textstyle\sum_{k=1}^{N}(A^*)^k)\Delta V(0) \quad (12)$$

where N is a number of basis factors.

Assumption 2. We consider that:

a) By virtue of the fact that cognitive model is very approximate, periodic correlation of its state with current state of situation is necessary. Such correlation is realized at times to be referred as correction ones (and be denoted by t with star);

b) Forced changes of coordinates of control vector are pulsed ones and are put in correction times only;

c) Between two correction times an influence from each factor is transferred to any factors connecting to it;

d) User changes his goals more rarely in comparison with distance between two adjacent times of correction.

Let $V(t^*)$ be corrected state of the model. Subject to (6)-(7) and (12) the vector $y^*(t^*+\Delta t)$ of goal coordinates at time $t^*+\Delta t$ is defined as

$$y(t^* + \Delta t) = C(t^*)V(t^*) + C(t^*)(\sum_{k=1}^{N}(A^*)^k)\Delta V(t^*) + C(t^*)(\sum_{k=0}^{N}(A^*)^k)Bg(t^*). \quad (13)$$

Let us denote

$$C(t^*)(\sum_{k=1}^{N}(A^*)^k) = W_1$$

$$C(t^*)(\sum_{k=0}^{N}(A^*)^k)B = W.$$

$$D(t^*) = y^*(t^*) - C(t^*)V(t^*) - C(t^*)W_1(t^*)\Delta V(t^*),$$

where $y^*(t)$ be the goal vector at time t^*. Subject to (13) we have

$$W(t^*)\, g(t^*) = D(t^*). \quad (14)$$

As in previous case, we find the solution of equation (14) with use of least squares method. The normal solution of (14) is

$$g(t^*) = W^+(t^*)\, D(t^*), \quad (15)$$

where $W^+(t^*)$ be the pseudoinverse matrix for matrix $W(t^*)$. The solution (15) indicates:

- which control factors are to be taken into account by the user at time t*;

- to what extent and in which direction the user should change each chosen factor to approach to his goal vector *y* (t*).*

According to note 2, this solution should be considered as approximate draft of user's actions to attain his preassigned goal y*(t*).

4. CONCLUSION

The analytical approach to the analysis and control of processes in situations with lack of the sufficient numerical information about their internal processes is offered. This approach may be applied for solution of economic, social, political, ecological, and other problems, taking into account that the construction of exact numerical models of complex social situations is hampered or even impossible by virtue of lack of required numerical information.

The approach represented herein was applied successfully to solution of problems of regional development, manufacturing development, etc. The results of these works may be seen on http://www.ipu.ru/labs/lab51/51_home.htm.

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